

Homework 2

Due on March 28th 2018 (at the beginning of class)

Problem I: Solve all the simple static games posted on BB in the separate file. We did some (or at least very similar games) in class, but make sure you have absolutely no problems understanding dominant strategies, and Nash equilibria.

Problem II: Consider the following simultaneous game (also known as the battle of the sexes): Bob and Jessica love each other and they like to spend time together. They are deciding to go on a date and they have to choose between going to a football game or going to opera. Bob likes football, while Jessica likes opera. Assume they receive a payoff of 3 if they end up going to their favorite event as long as they go together. If they go to their partner favorite event, but they are still going together they get a payoff of 1. If they end up going to any event by themselves, they both get 0.

a) Assume they make their decisions simultaneously. Write the game in its normal form representation and find all the Nash equilibria (both in pure and mixed strategies). $\mathbf{NE}_1 = \{\text{Football, Football}\}$, $\mathbf{NE}_2 = \{\text{Opera, Opera}\}$, $\mathbf{NE}_3 = \{\text{Bob : } P_{\text{Football}} = 3/4, P_{\text{Opera}} = 1/4; \text{ Jessica : } P_{\text{Football}} = 1/4, P_{\text{Opera}} = 3/4\}$

b) Now assume the game is played sequentially. Assume Jessica can leave a note on Bob's desk asking him to meet her at the football game or at the opera house. Later during the day, Bob reads the message then chooses where to go: to football or to opera. What will be the outcome in this case? How about if things are reversed and Bob is the one who leaves the note.

Problem III: Consider the following dynamic game: An entrant chooses whether to enter the market or stay out. If he chooses to stay out he will get \$0, while the incumbent gets \$20. If he enters the market, the entrant and the incumbent play the following simultaneous pricing game: they both choose whether to price high or low. If they both price low, they each get \$5. If they both price high, they each get \$10. If one prices low while the other one prices high, the one who prices low gets \$15 and the one who prices high gets \$0.

a) find the sub game perfect nash equilibrium of this game ($\mathbf{SPNE} = \{\text{Enter; Low Low}\}$)
b) find the equilibrium of a modified game where the entrant's outside option is no longer \$0, but \$10 ($\mathbf{SPNE} = \{\text{Stay Out; Low Low}\}$)

Problem IV: Write the rock-paper-scissors game in its normal form representation, and find its Nash Equilibrium (in mixed strategies, since there is no equilibrium in pure strategies for this game). (**All probabilities should be 1/3**)

Problem V: US is engaged in a military dispute with North Korea. Model this as a dynamic game with the US being the leader and NK being the follower (NK observes the US action and then acts). US has the option to attack NK or to not attack NK. After observing the US choice, NK has the option to attack or not. Assume any kind of retaliation infuriates the

US and leads to a full nuclear war which results in both countries getting a payoff of $-\infty$. On the other hand, if neither country attacks they both get 0. Finally, US is a bit angry at NK and would like at least to rough them up a little bit. Assume that the US attacks with conventional weapons and if NK doesn't retaliate, US gets a payoff of 10, while NK gets -10.

- a) Write this game in its extensive form and find the subgame perfect equilibrium (SPNE). (SPNE={**Attack; Don't Attack, Don't Attack**})
- b) Write the game in its normal form and solve for the Nash equilibria (NE). ($NE_1=\{\textbf{Attack; Don't Attack, Don't Attack}\}$, $NE_2=\{\textbf{Attack; Don't Attack, Attack}\}$, $NE_3=\{\textbf{Don't Attack; Attack, Don't Attack}\}$)

Problem VI: Consider a one shot Prisoner's Dilemma game where the competitive outcome (the Nash Equilibrium) gives both players \$10 each, the cooperative outcome gives \$20 each, and cheating when your opponent cooperates gives you \$30. The discount factor is δ .

- a) under what conditions can cooperation be sustained, if we assume players follow a GRIM strategy and the game is repeated every day for exactly 10 years in a row? (**Not sustainable**)
- b) under what conditions can cooperation be sustained, if we assume players follow a GRIM strategy and the game is repeated indefinitely? (**if $\delta > 1/2$**)
- c) repeat a, and b for other payoffs (10 each for the competitive outcome, 20 each for cooperation, but now 50 for defection). (**never for a and 3/4 for b**)

Problem VII: Two identical firms (identical cost functions) operate on a market. For each of the following market demand curves and cost curves determine the Bertrand, Cournot, and Stackelberg outcomes (prices, quantities, and profits - for each firm, and at the market level). Also determine the collusive outcome (assuming the two firms form a cartel). Compare the outcomes.

- a) $P = 200 - 2Q$, $TC = 50 + 10Q$ ($P_B = 10$, $P_C = 73.33$, $P_S = 57.5$, $P_M = 105$)
- b) $P = 250 - Q$, $TC = 50Q$ ($P_B = 50$, $P_C = 350/3$, $P_S = 100$, $P_M = 150$)
- c) $P = 1200 - Q$, $TC = 25 + 40Q$ ($P_B = 40$, $P_C = 426.67$, $P_S = 330$, $P_M = 620$)
- d) $P = 160 - 2Q$, $TC = Q^2$ ($P_B = 160/3$, $P_C = 80$, $P_S = 1600/21$, $P_M = 96$)

Problem VIII: Assume the market demand for a product is given by $Q^D = 840 - 2P$. There is one firm on this market with total cost $TC = 240 + Q^2$.

- a) Assuming this firm has a monopoly on this market, what will be the optimal price and quantity that this firm will choose? (**P=350, Q=140**)
- b) Assume that this firm is threatened by an outside firm. What would be the maximum amount of money that this firm would be willing to pay in order to preserve its monopoly power and not allow the market to become a 2 firms Bertrand competition? (**9800**)
- c) Assume now that there are two such firms with identical cost functions that compete in Cournot fashion on this market. What will be the resulting market outcome? (**P=300, Q=120 for each firm**)
- d) Assuming these two firms interact repeatedly and for an indefinite amount of time. Under what conditions can they sustain a cartel and under what conditions they will not be able to sustain the cartel and revert to Cournot competition. Assume the time discount factor is δ and both firms play a GRIM strategy. (**$\delta > 0.5052$**)